Realization of Multi-Band 3-dB Branch-Line Couplers Using Fourier-Based Transmission Line Profiles

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Published online: 24 Jan 2014.
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Abstract A new procedure for the design of multi-band branch-line couplers utilizing microstrip non-uniform transmission lines with Fourier-based profiles is presented in this article. The general method is accomplished by replacing the quarter-wave uniform transmission lines of a conventional coupler with multi-band non-uniform transmission lines of the same lengths, without including additional transmission lines or matching networks. The design of non-uniform transmission line couplers is performed using even- and odd-mode analysis. The proposed structure is relatively simple and is fabricated on a single-layered microstrip board. Two design examples of dual- and triple-band branch-line couplers suitable for global system mobile (GSM), wireless local area networks, and WiFi communications are designed, fabricated, and experimentally evaluated to validate the proposed methodology.

Keywords branch-line coupler, Fourier, microstrip, multi-band 3-dB couplers, non-uniform transmission line

1. Introduction

Microwave couplers are essential components for a host of system applications, such as modern radar, test equipment, and RF mixers, where reduced-size circuitry and multi-frequency operation are two main requirements. The quadrature branch-line coupler (BLC) is a well-known component that has been extensively addressed in the literature due to its wide use in microwave subsystems. However, the inherent single-frequency matching nature of conventional couplers that are based on quarter-wave transmission lines limits its applications to multi-band systems. Normally, dual-frequency
characteristics are achieved through the use of dual-band quarter-wavelength impedance transformers (Cheng & Yeung, 2012) attained by the proper selection of the circuit parameters. Nevertheless, the increase in the circuitry size is a major concern for such an approach and may not be an option for given design constraints.

Another way to realize the dual-band characteristics of BLCs is either by (a) using unequal arms lengths and a center-tapped stub (Park, 2009); (b) incorporating stepped-impedance stubs placed at the middle of each quarter-wave arm of the conventional coupler (Chin et al., 2010); (c) using four open-ended quarter-wave transmission lines at each port of the conventional BLC (Cheng & Wong, 2004), where the lengths of the additional stubs as well as the main arms are evaluated at the middle frequency of the two operating bands; or (d) employing three coupled-line sections, as demonstrated in Yeung (2011). Lin et al. (2010) proposed a tri-band BLC with three controllable operating frequencies employing four matching stubs at each port. A similar technique was developed in Tanigawa et al. (2007), for which triple-broadband matching techniques employing matching stubs were considered in the design of a 3-dB BLC. A tri-band BLC using double-Lorentz transmission lines was introduced in Lee and Nam (2007), where lumped capacitors and inductors were incorporated in the middle of each arm of a conventional BLC, and the design of a tri-band coupler for WiMAX applications was investigated in Sinchangreed et al. (2011).

The lack of detailed design procedure and analysis, however, makes the design of such multi-band couplers a difficult task. Recently, triple- and quad-band 3-dB couplers were proposed by adopting optimized compensation techniques to satisfy the matching conditions (Liou et al., 2009; Piazzon et al., 2012). It is worth pointing out that multi-band couplers in Cheng and Yeung (2012), Park (2009), Chin et al. (2010), Cheng and Wong (2004), Yeung (2011), Lin et al. (2010), Tanigawa et al. (2007), Sinchangreed et al. (2011), and Piazzon et al. (2012) were realized by adding extra transmission lines and/or matching stubs, which remarkably increases the overall circuit size; however, in Lee and Nam (2007), lumped elements were incorporated in the design. Typically, single-stage couplers exhibit narrowband frequency characteristics, and broader bandwidth is achieved by using cascaded couplers (Jung et al., 2009) or broadband matching techniques (Tanigawa et al., 2007) that are beyond the scope of this work.

In this article, equally split dual- and triple-band BLCs based on non-uniform transmission lines (NTLs) are proposed. Practical frequencies are chosen as design frequencies for two examples. In addition to the excellent electrical performance of the input port matching, isolation, transmission, and coupling characteristics at each design frequency in both couplers, no extra matching stubs are included in the design. As such, no extra circuit area is required.

The organization of this article is as follows: Section 2 presents the proposed step-by-step procedure of the multi-band 3-dB BLCs. Section 3 discusses the implementation of two design examples, including dual- and triple-band BLCs, and reports their measured performances, while conclusions are presented in Section 4.

2. Multi-Band 90° BLC Design

In this section, the theory of designing multi-band BLCs utilizing NTLs is presented. Figure 1 shows the proposed multi-band BLC, which consists of six variable profile impedances formed from \( Z_1(z) \) and \( Z_2(z) \) with lengths \( d_1 \) and \( d_2 \), respectively, while Figure 2 illustrates the corresponding even- and odd-mode circuits.
Referring to Figure 2, the total $ABCD$ matrix of the whole design can be found by multiplying the $ABCD$ parameters of each individual arm, described by:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_{\text{even}} = \begin{bmatrix}
1 & 0 \\
(Z_{\text{even}})^{-1} & 1
\end{bmatrix} \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} Z_{2(z)} \begin{bmatrix}
1 & 0 \\
(Z_{\text{even}})^{-1} & 1
\end{bmatrix}.
\] (1a)

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_{\text{odd}} = \begin{bmatrix}
1 & 0 \\
(Z_{\text{odd}})^{-1} & 1
\end{bmatrix} \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} Z_{2(z)} \begin{bmatrix}
1 & 0 \\
(Z_{\text{odd}})^{-1} & 1
\end{bmatrix}.
\] (1b)

The $ABCD$ parameters of the non-uniform impedance profile $Z_j(z)$ ($j = 1, 2$) can be determined by subdividing such an impedance into $K$ uniform electrically short segments,
each with length $\Delta z$. The $ABCD$ parameters of the overall arm $Z_j(z)$ are then obtained by multiplying the $ABCD$ parameters of each section as follows (Pozar, 2005):

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_{Z_j(z)} = \begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix} \cdots \begin{bmatrix}
A_i & B_i \\
C_i & D_i
\end{bmatrix} \cdots \begin{bmatrix}
A_K & B_K \\
C_K & D_K
\end{bmatrix}.
\]

(2)

where the $ABCD$ parameters of the $i$th segment are (assuming lossless transmission lines) (Pozar, 2005):

\[
A_i = D_i = \cos(\Delta \theta),
\]

(3a)

\[
B_i = Z^2((i - 0.5)\Delta z)C_i = jZ((i - 0.5)\Delta z) \sin(\Delta \theta), \quad i = 1, 2, \ldots, K,
\]

(3b)

\[
\Delta \theta = \frac{2\pi}{\lambda} \Delta z = \frac{2\pi}{c} f \sqrt{\varepsilon_{\text{eff}}} \Delta z.
\]

(3c)

The effective dielectric constant $\varepsilon_{\text{eff}}$ of each section is calculated using the well-known microstrip line formulas given in Pozar (2005). Then, the total uniformly subdivided microstrip sections that form the characteristic impedance $Z_j(z)$ are approximated by means of a truncated Fourier series with unknown coefficients (Khalaj-Amirhosseini, 2006; 2008):

\[
\ln \left( \frac{Z_j(z)}{Z_{o_j}} \right) = \sum_{n=0}^{N} C_n \cos \left( \frac{2\pi nz}{d_j} \right).
\]

(4)

The microstrip line lengths $d_1$ and $d_2$ are chosen to be $\lambda/8$ and $\lambda/4$, respectively, at the lowest design frequency $f_1$. These wavelengths are those corresponding to uniform microstrip lines with impedances of 50 and 35 $\Omega$. The characteristic impedances, $Z_{o1}$ and $Z_{o2}$, are equal to 50 and 35 $\Omega$, respectively, and $N$ is the number of the Fourier series coefficients. Moreover, $Z_j(z)$ must be restricted by constraints that allow practical realization and easy fabrication for the proposed couplers, as follows:

\[
Z_{\text{min}} \leq Z_j(z) \leq Z_{\text{max}}.
\]

(5a)

\[
Z_j(0) = Z_j(d_j) = 1.
\]

(5b)

Upon determining the $ABCD$ parameters of $Z_1(z)$ by following the design equations (Eqs. (2) through (5b)), the following equation can be written as:

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_{Z_1(z)} \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}.
\]

(6)

Setting $I_2$ in Eq. (6) to zero, and solving for $\frac{V_1}{I_1}$, one obtains:

\[
\frac{V_1}{I_1} = \frac{A}{C} = Z_{\text{even}}.
\]

(7)
Similarly, the odd-mode input impedance $Z_{in}^{odd}$ shown in Figure 2(b) can be determined by setting $V_2$ in Eq. (6) to zero, leading to:

$$\frac{V_1}{I_1} = B \frac{1}{D} = Z_{in}^{odd}.$$  \hspace{1cm} (8)

Consequently, the $ABCD$ matrices for the circuit modes shown in Figure 2 are calculated using Eqs. (1a) and (1b), where the non-uniform impedance $Z_2(z)$ is determined by following the design equations (Eqs. (2) through (5b)). Thus, the total input impedance for each mode can be expressed by the following equation (Pozar, 2005):

$$Z_{in,e}^{total} = \frac{A_{e,o} Z_o + B_{e,o}}{C_{e,o} Z_o + D_{e,o}},$$  \hspace{1cm} (9)

where $Z_o$ is the characteristic impedance of each feed port. Here, the procedure carried out in Eqs. (1a) to (9) is followed for each design frequency $f_m$ ($m = 1, 2, \ldots, M$). Thus, the reflection and transmission coefficients for the NTL BLC can be written as (Pozar, 2005):

$$\Gamma_{e,o}(f_m) = \frac{Z_{in,e}^{total} - Z_o}{Z_{in,e}^{total} + Z_o},$$  \hspace{1cm} (10a)

$$T_{e,o}(f_m) = \frac{2}{A_{e,o} + \frac{B_{e,o}}{Z_o} + C_{e,o} Z_o + D_{e,o}}.$$  \hspace{1cm} (10b)

The $S$-parameters for the NTL BLC can be calculated using the following equations (Pozar, 2005):

$$S_{11}(f_m) = \frac{\Gamma_e(f_m) + \Gamma_o(f_m)}{2},$$  \hspace{1cm} (11a)

$$S_{41}(f_m) = \frac{\Gamma_e(f_m) - \Gamma_o(f_m)}{2},$$  \hspace{1cm} (11b)

$$S_{21}(f_m) = \frac{T_e(f_m) + T_o(f_m)}{2},$$  \hspace{1cm} (11c)

$$S_{31}(f_m) = \frac{T_E(f_m) - T_o(f_m)}{2}.$$  \hspace{1cm} (11d)

Finally, to obtain the desired response at the design frequencies, the optimum values of the Fourier coefficients $C_n$ can be obtained through minimizing the following error function:

$$E(f_1, \ldots, f_M) = \sqrt{\frac{|S_{11}|^2 + |S_{41}|^2 + |S_{21}|^2 - |S_{21}|^2_{des} + |S_{31}|^2 - |S_{31}|^2_{des}}}{16M},$$  \hspace{1cm} (12)

where $|S_{21}|_{des} = |S_{31}|_{des} = 0.707$, and $M$ corresponds to the number of design frequencies. The term $16M$ in the denominator acts as a normalization factor; the error function in Eq. (12) can be formulated in several ways to achieve the design goals. To solve such a constrained optimization problem, a trust-region-reflective algorithm (Li,
1993) is adopted that can solve large-scale bound-constrained non-linear minimization problems with strong convergence properties. Figure 3 shows a flowchart summarizing the design procedure described by Eqs. (1) through (12). In the optimization process, there is no unique solution for the unknown Fourier coefficients values, and each optimization run results in a different set. However, the one that gives the optimal response accompanied with an impedance profile that is realizable and complies with Eqs. (5a) and (5b) is considered in the further design steps. Furthermore, the design principle can be

![Figure 3. Flowchart demonstrating general design procedure of multi-band non-uniform BLC.](image)
generalized to design BLCs of an unequal split type, taking into account proper values of the branches impedances (see Rawat et al. [2013] and the references therein).

3. Design Examples

Two design examples of dual- and triple-band BLCs are presented in this section to validate the design procedure proposed in Section 2. For justification, the proposed designs are simulated using two different full-wave electromagnetic simulators, namely IE3D (Mentor Graphics PCB Design Software, 2006) and HFSS (Ansys, Inc., 2011), which are based on the method of moments (MoM) and the finite-element method (FEM), respectively. For further validation, the presented examples are also fabricated and tested considering a Rogers RO4835 substrate with a relative permittivity of 3.48, loss tangent of 0.0037, substrate thickness of 1.524 mm, and copper clad thickness of 17 μm.

3.1. Dual-Band 3-dB BLC

Considering the design steps demonstrated in Figure 3, a dual-frequency NTL BLC with design frequencies chosen to be 0.9 and 2.4 GHz is presented. The 0.9 GHz frequency band is widely used in global system mobile (GSM) technology, whereas the 2.4 GHz band fits in many wireless applications, such as IEEE 802.11b,g,n standards (wireless local area network [WLAN] and/or WiFi). In the dual-band BLC example, two NTLs—$Z_1(z)$ and $Z_2(z)$, with widths $2.5 \, \text{mm} < W_1(z) < 5 \, \text{mm}$ and $2 \, \text{mm} < W_2(z) < 12 \, \text{mm}$, respectively—are designed with lengths $d_1$ and $d_2$ of 25.18 and 49.27 mm, respectively, whereas the characteristic impedances $Z_{o1}$, $Z_{o2}$, and $Z_o$ are chosen to be 50, 35, and 50 Ω, respectively. $K_1$, $K_2$, and $N$ are set to 50, 50, and 10, respectively, and the resulting error value from the optimization process was 0.022. Table 1 shows the obtained Fourier series coefficients for $Z_1(z)$ and $Z_2(z)$.

<table>
<thead>
<tr>
<th>Fourier coefficients for $Z_1(z)$</th>
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<tbody>
<tr>
<td>$C_0$</td>
</tr>
<tr>
<td>0.0534</td>
</tr>
<tr>
<td>$C_6$</td>
</tr>
<tr>
<td>0.0036</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fourier coefficients for $Z_2(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
</tr>
<tr>
<td>-0.0370</td>
</tr>
<tr>
<td>$C_6$</td>
</tr>
<tr>
<td>0.0193</td>
</tr>
</tbody>
</table>
Figure 4 shows a photograph of the fabricated dual-band coupler, while Figure 5 shows the full-wave simulation results. As can be seen from Figure 5(a), simulation results indicate that the input port matching parameter ($S_{11}$) is below $-20$ and $-18$ dB at 0.83 and 2.4 GHz, respectively, and the obtained experimental results using an Agilent E5071B vector network analyzer (VNA) are in good agreement with the resulting simulations.

Figure 5. Simulation and measurement results for dual-band 3-dB 90° BLC: (a) input port matching parameter ($S_{11}$), (b) isolation parameter ($S_{41}$), (c) transmission parameter ($S_{21}$), and (d) transmission parameter ($S_{31}$).
Moreover, the isolation parameter $S_{41}$ is below $-20$ dB at the design frequencies, as shown in Figure 5(b). The simulated transmission parameter $S_{21}$, shown in Figure 5(c), is equal to $-2.9$ and $-2.7$ dB at the first and second bands, respectively, which are very close to the theoretical value of $-3$ dB, whereas the results obtained from measurement are around $-3.4$ dB. Furthermore, Figure 5(d) shows the simulated transmission parameter $S_{31}$, which equals $-3.4$ dB at 0.9 and 2.4 GHz. Those values are also close to $-3$ dB, whereas the measured results are $-3.5$ dB in proximity to the two design frequencies. It is worth pointing out here that the slight discrepancies between the simulated and measured results are thought to be due to connector losses as well as measurement errors.

Figure 6 shows the magnitude of the measured phase difference between the transmission parameters, $S_{21}$ and $S_{31}$. As expected, a phase difference of $90\cdot5$ can be clearly discerned at the two design frequencies, 0.9 and 2.4 GHz.

Figure 7 illustrates the simulated current distributions at an arbitrarily selected in-band frequency of 0.86 GHz and out-of-band frequency of 1.5 GHz. It can be seen that at the in-band frequency (Figure 7(a)), there are weak current distributions at port 4, which confirms the proper operation of the proposed coupler as the isolation port possesses
high impedance at such a frequency. On the contrary, relatively higher electric currents are concentrated along the isolation port at the out-of-band frequency (Figure 7(b)).

### 3.2. Triple-Band 3-dB BLC

After the successful implementation of the dual-band NTL BLC, a triple-band coupler was implemented in a similar fashion to prove the validity, repeatability, and robustness of the underlying design procedure. The proposed triple-band NTL BLC is designed to operate at three concurrent frequencies, specifically, 0.9, 2.4, and 5.4 GHz. Such bands find many applications in modern wireless communications, such as GSM, WLAN, WiFi, and WiMAX technologies. Considering the same previously mentioned substrate, two NTLs—$Z_1(z)$ and $Z_2(z)$ with widths $1 \text{ mm} < W_1(z) < 5.5 \text{ mm}$ and $1 \text{ mm} < W_2(z) < 10 \text{ mm}$, respectively—are designed with lengths $d_1$ and $d_2$ of 25.18 and 49.27 mm, respectively, which equals $\lambda/8$ and $\lambda/4$ at the lowest design frequency (i.e., 0.9 GHz), whereas the characteristic impedances $Z_{o1}$, $Z_{o2}$, and $Z_o$ are chosen to be 50, 35, and 50 $\Omega$, respectively. Similar to the dual-band design, $K_1$, $K_2$, and $N$ are set to 50, 50, and 10, respectively. The error value from the optimization process was 0.026. Table 2 shows the obtained Fourier series coefficients for $Z_1(z)$ and $Z_2(z)$.

Figure 8 shows a photograph of the implemented triple-band BLC, whereas Figure 9 represents the simulated and measured response of the corresponding $S$-parameters. As can be seen from Figures 9(a) and 9(b), the simulated input matching and isolation parameters are both below $-20$ dB at the design frequencies, and the measured results are in very good agreement with the simulated ones. Moreover, the simulated transmission parameters ($S_{21}$ and $S_{31}$) are in the ranges of $-2.5$ to $-3.5$ dB, $-2.5$ to $-3.6$ dB, and $-3.4$ to $-4$ dB at 0.86, 2.44, and 5.48 GHz, respectively. Furthermore, measured results in the ranges of $-3.1$ to $-3.6$ dB, $-3.4$ to $-4.5$ dB, and $-4.3$ to $-4.6$ dB were obtained at the first, second, and third bands, respectively. The slight frequency shifts, as well as

| Fourier coefficients for $Z_1(z)$ |
|---|---|---|---|---|---|
| $C_0$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
| 0.1782 | 0.1954 | 0.2223 | $-0.2163$ | $-0.0881$ | $-0.0589$ |
| $C_6$ | $C_7$ | $C_8$ | $C_9$ | $C_{10}$ |
| $-0.0540$ | $-0.0487$ | $-0.0450$ | $-0.0432$ | $-0.0418$ |

| Fourier coefficients for $Z_2(z)$ |
|---|---|---|---|---|---|
| $C_0$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
| 0.1680 | 0.1112 | 0.3284 | $-0.0909$ | 0.2555 | $-0.2732$ |
| $C_6$ | $C_7$ | $C_8$ | $C_9$ | $C_{10}$ |
| $-0.1565$ | $-0.0920$ | $-0.0933$ | $-0.0783$ | $-0.0790$ |
the increased losses, are thought to be due to the resulting optimization error, different
types of losses, and measurement errors.

Figure 10 illustrates the simulated and measured phase difference between the two
output ports. It can be clearly seen that a phase shift of $90 \pm 7^\circ$ occurs around the design
frequencies (i.e., 0.9, 2.4, and 5.4 GHz).

Figure 9. Simulation and measured results for triple-band 3-dB $90^\circ$ BLC: (a) input port matching
parameter ($S_{11}$), (b) isolation parameter ($S_{41}$), (c) transmission parameter ($S_{21}$), and (d) transmission
parameter ($S_{31}$).
4. Conclusions

In this article, a new approach for the design of multi-band BLCs is investigated to overcome realization difficulties that are often encountered with conventional BLCs. Based on the NTL theory, an efficient technique for the design of multi-frequency 3-dB BLCs is proposed. Each uniform transmission line in the conventional BLC is replaced with a single NTL exhibiting a Fourier-based profile. Even- and odd-mode analysis and an optimization-driven process are carried out to achieve the desired response at the design frequencies. To justify the design principle, dual- and triple-band BLCs suitable for various modern wireless applications were designed, fabricated, and measured. The agreement between both simulated and measured results proves the design methodology.

Funding

This work has been supported by the National Science Foundation through an Engineering Design and Innovation grant (no. 1000744).

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